**Integrality/Separability of Number and Cumulative Area in Pigeons**

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Numerical cognition is part of our daily life and a vital skill for adaptation and survival. Whether it is to choose the shorter line at the grocery store or calculate the time it will take to get to an appointment on time, numerical estimations are needed. Although some computations require a formal system of rules and symbols that allow precise numerical representations (Nieder, 2005), simpler estimations can be achieved with non-symbolic representations of number (Karolis and Butterworth, 2016). For example, studies in pre-verbal numerical cognition have shown that 6-month-old infants can discriminate relatively large sets of items (Xu and Spelke, 2000), and studies that limit adult participants’ opportunity to count, have found that they are still capable of making approximate estimations of numerosity (Whalen, Gallistel, and Gelman, 1999; Adriano, Girelli, and Rinaldi, 2021).

Moreover, numerical abilities are not exclusive to humans: chimpanzees (Beran, 2001; Beran 2004) and rhesus monkeys (Hauser, Carey and Hauser, 2000) choose containers with more food after seeing two different containers being filled, single mosquitofish join larger shoals when given the choice between two shoals (Agrillo, Dadda, Serena and Bisazza, 2008), and even honeybees spontaneously choose patches that contain more flowers that have been previously associated with sucrose availability (Howard et al, 2020).

A hallmark of these non-symbolic numerical representations is that they are imprecise and decrease in accuracy as the estimated quantities increase. These patterns follow Weber’s Law, a psychophysical law that states that the perceived change in magnitude between two stimuli depends on the proportion of the magnitudes, rather than their absolute value of change (Cantlon and Brannon, 2006; Feigenson, Dehaene and Spelke, 2004). A consequence of these estimations are two behavioral signatures: the distance effect and the size effect. The distance effect is observed when more distant quantities are easier to discriminate than closer quantities, for example it is easier to discriminate 2 from 10 items than it is to discriminate 4 from 6. On the other hand, the size effect is observed when it is easier to discriminate smaller quantities than bigger ones (even if the absolute difference is the same), for example it is easier to discriminate 2 from 3 than it is to discriminate 7 from 8.

The prevalence of the ability to estimate numerosities across species suggests the existence of an ancient mechanism that has been preserved through evolution, similar to a ‘sense of number’. One of the most prevalent accounts for this mechanism is the Approximate Number System (ANS), which states a specialized mechanism is dedicated to processing numerical information from stimuli (Starr and Brannon, 2015). The ANS allows quick, approximate estimations of numerical information without the need for symbolic representations and has been found in humans (Xu and Spelke, 2000; Pica, Lemer, Izard, and Dehaene, 2004), non-human primates (Cantlon and Brannon, 2006), birds (Scarf, Hayne, and Colombo, 2011), rats (Meck and Church, 1983) and other species (Agrillo, Piffer, and Bisazza, 2010; Gazzola, Vallortigara, and Pellitteri-Rosa, 2018).

Usually, when we make a numerical judgment, we have access to multiple sources of information that can contribute to the estimation. For example, to estimate the shortest queue at the grocery store, one could count all the people standing in each queue, or one could consider the cumulative surface area of each queue, which is likely to correlate with the number of people. In nature, these kinds of non-numerical attributes, such as cumulative space occupied or total area of the stimuli, usually covary. In fact, studies exploring the effects that perceptual features have in numerical representations have found continuous properties of the displays like cumulative surface area (Yousif and Keil, 2020), density (Chakravarthi and Bertamini, 2020), and variability (DeWind, Bonner, and Brannon, 2020) influence numerical judgments. From the ANS perspective, these interactions are the result of independent representations of numerical and non-numerical properties that are later used by domain-general mechanisms that are involved in the judgment process (Cantlon, Platt and Brannon, 2009; Cohen Kadosh, Gevers and Notebaert, 2011).

Although the ANS has been widely accepted as the mechanism through which organisms extract numerical information from displays, other accounts advocate for more global representations of magnitude (Leibovich, et al., 2017). One of these alternative accounts is the General Magnitude System (GMS), which proposes that all magnitude estimations, both numerical and non-numerical, are controlled by a common mechanism (Lourenco, 2015; Lourenco and Aulet, 2019). From the GMS perspective, numerical representations emerge from non-numerical magnitude representations, implying that they are not independent from each other. This model is supported by interactions between space and number estimations (Drucker and Brannon, 2014), and studies that indicate that numerical and non-numerical representations activate similar neural mechanisms (Nieder, 2011; Ditz and Nieder, 2015).

The relationship between numerical and non-numerical representations is especially relevant in the context of these two contending theoretical accounts. Thus, the assessment of integrality and separability of numerical and non-numerical representations offers relevant insight to this question. Integrality is found when a change in one dimension causes a perceived change in the other dimension, indicating a common representation mechanism. Alternatively, separability is found when changes in one dimension do not result in perceived change in the other dimension, indicating separate representations. The multidimensional stimulus generalization model (Soto and Wasserman, 2010; Aulet and Lourenco, 2020) allows to differentiate between integral and separable dimensions by contrasting two distance metrics in a stimulus space defined by the two dimensions. If the dimensions are integral, the similarity between two stimuli in this space will be better described by the Euclidean distance metric. On the other hand, if two dimensions are separable, the similarity of two stimuli will be better described by the sum of the distance in each dimension which is known as the City-Block distance metric.

The present project aims to evaluate the relationship between cumulative area and number perception using the multidimensional stimulus generalization model. Here focus on the relationship between number and cumulative area because of the well-established interaction between these dimensions (Hurewitz, Gelman, and Schnitzer, 2006; Kubo, 2020). We created a stimulus space comprised of all combinations between nine levels of numerosity and nine levels of cumulative area. If number and cumulative area are perceived as integral dimensions, then the similarity between stimuli will be best described by the Euclidean distance metric. On the other hand, if number and cumulative area are perceived as separable dimensions, their relationship will be better described by the City-Block distance metric. We compare the relative fit of statistical models using both distance metrics to predict the generalization gradients obtained after training 4 pigeons in a two-alternative forced choice (2AFC) task for 40 sessions. Pigeons were trained to peck one of the stimuli on the stimulus space (S+), defined by a specific combination of number and cumulative area, while pecks to the remaining 80 stimuli were not reinforced (S-).

Although, the use of Bayesian analysis techniques is relatively new in this field (Martin, Wiener, and van Wassenhove, 2017), they can be useful to analyze the data in a more flexible way. It would be of interest to carry out more intuitive analyses, reviewing the contributions that the use of prior distributions and the relaxation of assumptions (such as homoscedasticity of variance in least squares or the multivariate normal distribution in maximum likelihood models) would make on the estimation of results and parameters. Moreover, this type of analysis will reflect truly probabilistic statements, analyzing the EAP’s, credible intervals and absolute and relative fit indices of the models (sampled from the posterior distribution).

Although simple, the two models that will be analyzed in this project represent two contemporary approaches for the representation of numerical and non-numerical magnitudes in humans and non-human animals. The comparison of relative fit between two different distance metrics of number and cumulative area will inform about the relationship between the dimensions: if the Euclidean model has a better fit, thus we could assume that pigeons’ perception of numerosity and cumulative area is integral; on the contrary, if the City-Block model achieves a better fit, then we could assume that both dimensions are perceived as separable.

Additionally, as pigeons learning to respond to trials takes time, there are at least three possible analysis strategies to consider. First, a general analysis could be conducted using all data from the training. A second alternative is to divide the training data into two parts as pigeons’ attentional strategies could change with learning. Finally, it is possible to evaluate integrality and separability using only data from later stages of training once learning is already stabilized. These and other decisions will be discussed in subsequent reports.

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